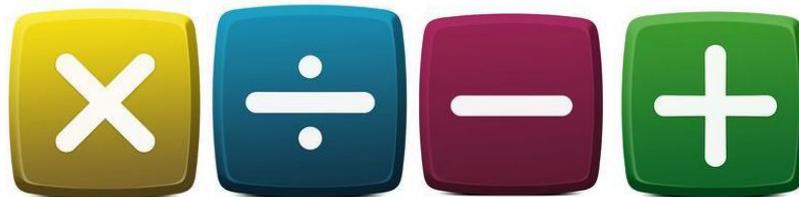




Calculation Policy



Examples of calculation methods for each year group and the progression between each method.

September 2015

Whole School Approach to Calculation

Introduction

This policy outlines the key stages of calculation throughout the school to meet the expectations of the new national curriculum but most importantly, the learning styles of the children at Redwell Primary School. The new curriculum focuses on skills and mastery and is not about moving children onto the next method as soon as they can do the one before. Working on more complex and richer problems, rather than new methods, will support the 'mastering' of Maths. We promote the use of the 'most efficient method' to solve a calculation.

Mental strategies

The written methods in this document are important but do not replace the wide range of mental calculation strategies taught. Children will be given opportunities to use and apply a wide range of mental calculations to solve a variety of 'real life' problems. They will be encouraged to use the most appropriate and efficient strategy for the numbers involved.

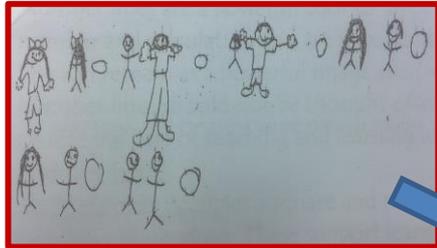
Progression in all calculation will be, wherever possible, taught in context;

- Practically
- Symbolically
- Using informal jottings
- Using number lines
- Vertically

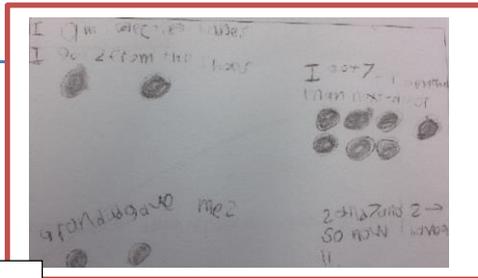
Using approximation and checking strategies

Progression in recording

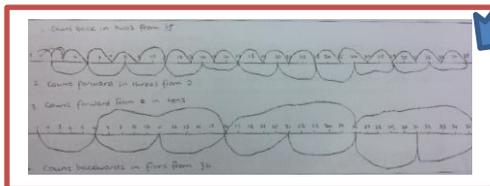
E.g. solving word problems using jottings. There are 12 children in the class. The children work in pairs in P.E. each pair has a ball. How many balls will I need?



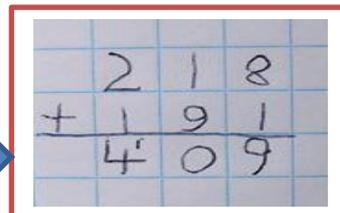
Initially children will record in their own way using pictorial representations.



Then recording will become more symbolic.



Children move into more abstract visual imagery such as number lines.



Then towards more formal written methods

Early Algebra

Children need to see number sentences written in different formats to help **recognise the equals sign as a 'balancing tool'**. Therefore, children will be given access to problems such as $12 = 4 \times \square$ and $8 + 4 = \square + 5$ and be encouraged to seek patterns. Progression in solving problems such as these will be by mixing operations within the number sentence then moving on to substituting the box with a letter in upper key stage 2.

Calculations.

Children need to be able to compute competently at all levels and explain what they are doing and why it works.

However, there are times e.g. in problem solving, where use of a calculator is appropriate. This may be for several reasons.

1. The child knows the type of calculation that is required, but the numbers are too large or complex and this will inhibit the chances of arriving at the correct solution.
2. In a trial and improvement situation where repeated calculations are necessary but demotivating to be done mechanically.
3. Time is short and the calculator is, therefore, an efficient use of time.
4. As a proof or checking tool.
5. When needing to handle large amounts of numbers efficiently e.g. adding lists.
Children should be encouraged to use calculators from Reception, where they can be used in play situations for number recognition, onwards.

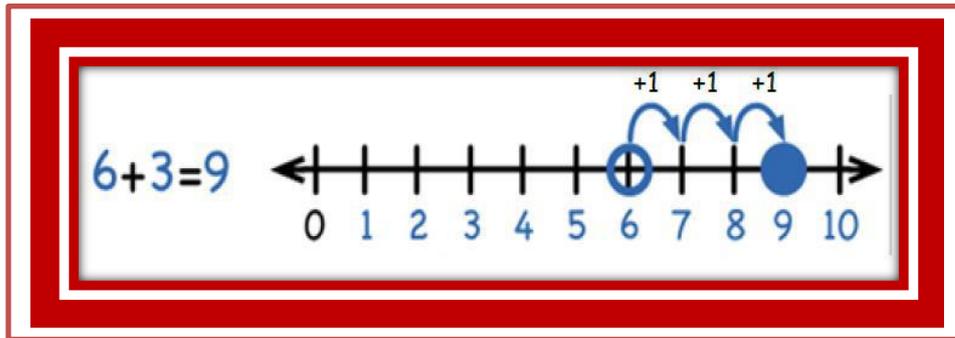
The more familiar children are with calculators the greater their confidence and the better their use of them will be. Children need to be taught when it is appropriate to use calculators and should be able to choose when they use them as long as their reason is justifiable. Children should not use calculators instead of learning written methods of calculation.

Addition

Year One

Focus: Adding with numbers up to 20

Children should use number lines (with the numbers on) to add by counting in ones. Starting with the greatest number and counting on the smaller number.



As well as using a numberline, children in Year 1 need to:

Use a variety of equipment to solve addition problems, including counting equipment, everyday objects, number tracks etc.

Read and write the addition (+) and equals (=) sign and use them in number sentences.

Solve addition number sentences and missing number problems: $7 + 4 = ?$, $1 + 2 + 1 = ?$, $? + ? = 9$ etc.



Key Vocabulary

Add, more, plus, and, make, altogether, total, equal to, equals, the same as, double, most, count on, numberline

Key Skills

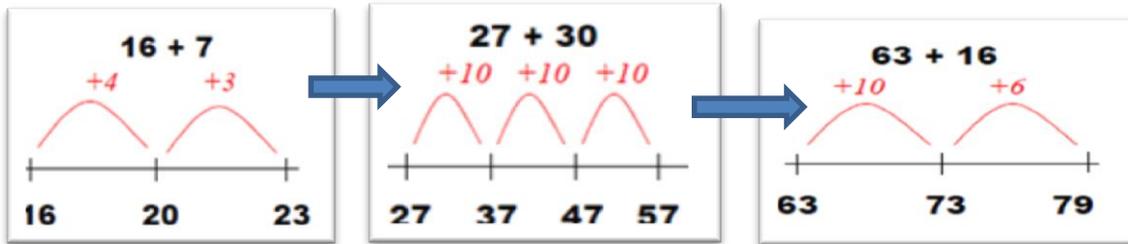
- Reading and writing numbers to 100 in numerals.
- Writing numbers to 20 in words including correct spelling.
- Counting to and across 100 in ones.
- Counting in multiples of 2, 5 and 10.
- Solving simple one step addition problems: using objects, numberlines and images to support.

Addition

Year Two

Focus: Adding with 2 digit numbers.

Children should explore and understand how to use blank numberlines to add using their knowledge of place value and how to partition numbers in different ways. Once confident they should move onto written partitioning methods.



Adding a 2 digit number and units

Adding a 2 digit number and tens

Adding two 2 digit numbers

$$\begin{array}{l} 34 + 23 = 57 \\ 30 + 20 = 50 \\ 4 + 3 = 7 \end{array}$$

$$\begin{array}{l} 58 + 43 = 101 \\ 50 + 40 = 90 \\ 8 + 3 = 11 \end{array}$$

$$\begin{array}{l} 78 + 47 = 125 \\ 70 + 40 = 110 \\ 8 + 7 = 15 \end{array}$$

Partitioning should be started with 2 digits numbers that do not bridge the tens or hundreds so children become fully confident with the method itself

Once children are confident they can start using the partitioning method to add numbers that bridge the tens and hundreds boundaries.

Key Vocabulary

Add, more, plus, and, make, altogether, total, equal to, equals, the same as, double, most, count on, numberline, *sum*, *tens*, *units*, *partition*, *addition*, *column*, *tens boundary*

Key Skills

Add a 2 digit number and units and a 2 digit number and 10s.

Add pairs of 2 digit numbers.

Add three single digit number.

Know and show that adding can be done in any order (the commutative law).

Recall bonds to 20 and multiple of 10 bonds to 100.

Count in steps of 2,3 and 5 and count in 10s from any number.

Understand the place value of 2 –digit numbers (tens and ones).

Compare and order numbers to 100 using < > and = signs.

Read and write numbers to at least 100 in numerals and words.

Solve contextual addition problems.

Addition

Year Three

Focus: Adding with numbers up to 3 digits

In year 3 we will move to the traditional column method and to support this, children will first apply their partitioning skills to the partitioning column method.

$$246 + 132 = 378$$

$$200 + 40 + 6$$

$$100 + 30 + 2$$

$$300 + 70 + 8 =$$

Introduce the partitioning column method with numbers that do not bridge so children become confident with the method itself.

Add units first!

$$337 + 188 = 525$$

$$300 + 30 + 7$$

$$100 + 80 + 8$$

$$400 + 110 + 15 = 525$$

Once confident, children can start using the partitioning column method to solve problems that bridge the tens and hundreds boundaries.

$$116 + 343 = 459$$

$$\begin{array}{r} 343 \\ + 116 \\ \hline 459 \end{array}$$

Now children are ready to move on to the traditional column methods. Introduce this initially with numbers that do not bridge any boundaries. It is important children remember that it is three hundred add one hundred, NOT 3 + 1!

$$245 + 84 = 329$$

$$\begin{array}{r} 245 \\ + 84 \\ \hline 329 \\ 1 \end{array}$$

Once the method is secure children are now ready to be introduced to 'carrying' which happens when bridging in the column method. Make sure children add the units first and 'carry' numbers under the bottom line.

Key Vocabulary

Add, more, plus, and, make, altogether, total, equal to, equals, the same as, double, most, count on, numberline, sum, tens, units, partition, addition, column, tens boundary, *hundreds boundary*, *increase*, *vertical*, *carry*, *expanded*, *compact*

Key Skills

- Read and write numbers to 1000 in numerals and words.
- Add 2 digit number mentally including those that bridge 100.
- Add a 3 digit number and ones, a 3 digit number and 10s and a 3 digit number and 100s mentally.
- Estimate answers to calculations, using the inverse operation to check.
- Solve problems, including missing number problems using number facts and place value.
- Recognise the place value of each digit in a 3 digit number (hundreds, tens and units).
- Continue to practice many different mental addition strategies including adding to the nearest multiple of 10, 100, 1000 and adjusting, using number bonds, using near doubles, partitioning and recombining etc.

Addition

Year Four

Focus: Adding with numbers up to 4 digits

In year 4 children will consolidate their use of the traditional column method and will be able to use it confidently to add numbers up to 4 digits. This could include carrying units, tens and hundreds.

$$4267 + 1584 = 5851$$

$$\begin{array}{r} 4267 \\ + 1584 \\ \hline 5851 \\ 11 \end{array}$$

Children should already be familiar with the column method from year 3 but it is very important to go over the method again ensuring children understand why they start with the units, have to carry a number etc.

Remember!

- 1) The units must be added first!
- 2) 'Carry' numbers underneath the bottom line!
- 3) Reinforce the place value! It is not 6 add 8, it is 6 tens add 8 tens!

Key Vocabulary

Add, more, plus, and, make, altogether, total, equal to, equals, the same as, double, most, count on, numberline, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, carry, expanded, compact, *thousands, hundreds, digits, inverse.*

Key Skills

- Select most appropriate method: mental, jottings or written and explain why.
- Recognise the place value of every digit in a 4 digit number.
- Round any number to the nearest 10, 100 or 1000.
- Estimate and use inverse operations to check answers.
- Solve 2 step problems in different contexts, picking the correct operation to use.
- Find 100 more or less than a number.
- Continue to use a wide range of mental addition methods.
- Add numbers with up to 4 digits using column addition.

Addition

Year Five

Focus: Adding with more than 4 digits

In year 5 children will now use the column method to add decimal numbers in the context of money and measures. It is important that children have place value skills beyond 4 digits here and fully understand what a decimal number represents.

The decimal point needs to be lined up just like all of the other place value columns and must be remembered in the answer column. It is important children understand why this is and get into this habit very quickly.

Children should be working with numbers greater than 4 digits including numbers in the ten thousands and hundred thousands.

$$\begin{array}{r} \text{£} 23.59 \\ + \text{£} 7.55 \\ \hline \text{£} 31.14 \end{array}$$

$$\begin{array}{r} 23481 \\ + 1362 \\ \hline 24843 \end{array}$$

Remember!

- 1) It is important that children say 6 tenths add 7 tenths so they understand that they are adding part of a number not a whole number.
- 2) Empty places should be filled with a zero to show the value of that place.

$$\begin{array}{r} 19.01 \\ 3.65 \\ + 0.70 \\ \hline 23.36 \end{array}$$

Children need to start using the column method to add more than two values, still considering place value very carefully.

Key Vocabulary

Add, more, plus, and, make, altogether, total, equal to, equals, the same as, double, most, count on, numberline, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, carry, expanded, compact, thousands, hundreds, digits, inverse, *decimal place*, *decimal point*, *tenths*, *hundredths*, *thousandths*.

Key Skills

- Add increasingly large numbers mentally using an expanding range of strategies.
- Use rounding to check answers and make estimates.
- Understand the place value of tenths and hundredths.
- Solve multi step problems in different contexts, deciding which operations and methods to use and explaining why.
- Read, write, order and compare number to 1 million.
- Round any number to 1 million to the nearest 10, 100, 1000, 10 000 or 100 000.
- Add numbers with more than 4 digits using column addition.

Addition

Year Six

Focus: Adding several numbers with an increasing level of complexity

In year 6 children need to use all the previous adding skills developed to add several numbers with a variety of different decimal places. Many of these problems will be in the context of money or measures.

Children need to use their knowledge of the decimal point to line up their amounts correctly in the column. Zeroes should be added to support place value, showing that there is no value to add.

Children should also continue to add multiple integers with 4 digits or more.

$$\begin{array}{r} 23.361 \\ 9.080 \\ 59.770 \\ + 1.300 \\ \hline 93.511 \\ \small 2 \quad 1 \quad 2 \end{array}$$

$$\begin{array}{r} 81,059 \\ 3,668 \\ 15,301 \\ + 20,551 \\ \hline 120,579 \\ \small 1 \quad 1 \quad 1 \quad 1 \end{array}$$

Key Vocabulary

Add, more, plus, and, make, altogether, total, equal to, equals, the same as, double, most, count on, numberline, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, carry, expanded, compact, thousands, hundreds, digits, inverse, decimal place, decimal point, tenths, hundredths, thousandths, *integer*

Key Skills

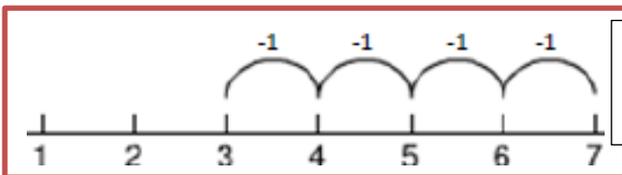
- Solve problems mentally, including those with mixed operations and large numbers, using all the mental strategies learnt in previous years.
- Solve multi step problems in context, deciding which operations and methods to use,
- Use estimation to check answers to a calculation.
- Read, write order and compare numbers to 10 million and understand the value of each digit.
- Round any whole number to the nearest 10, 100, 1000, 10 000, 100 000, 1 000 000 or 10 000 000
- Round decimal numbers to the nearest whole number.

Subtraction

Year One

Focus: Subtracting from numbers up to 20

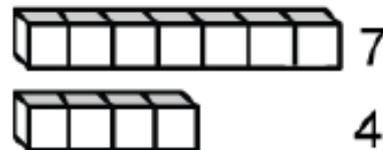
In year 1, children will use numberlines, objects and visual models to understand subtraction as taking away but also as the difference between or distance between two numbers.



For 7 take away 4, the child would start on the numberline at 7 and count back 4 in ones. This would give them the answer 3

Hundred squares, number tracks, counting objects and real life objects should all be used as well to explore subtraction in a variety of practical contexts.

To answer problems such as how many more is 7 than 4 or what is the difference between 7 and 4, cubes should be made into rods so children can see the problem visually. This method can also be used to answer 'find the distance' problems.



Mental subtraction is equally important and children should practice recalling subtraction facts up to and within 10 and 20. In year 1 children should also be taught about subtracting zero.

Key Vocabulary

Equal to, take, take away, less, minus, subtract, leaves, distance between, difference between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is_?

Key Skills

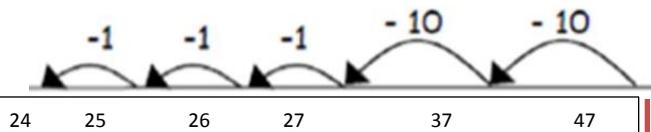
- Given a number, say one more or one less.
- Count to and over 100, forward and back from any number in 1s.
- Represent and use subtraction facts to 20 and within 20.
- Subtract with one digit and 2 digit numbers to 20, including zero.
- Solve one step problems that involve subtraction using objects, pictures and numbered lines.
- Read and write numbers to 100 in numerals.
- Write numbers in words to 20s, including correct spelling.

Subtraction

Year Two

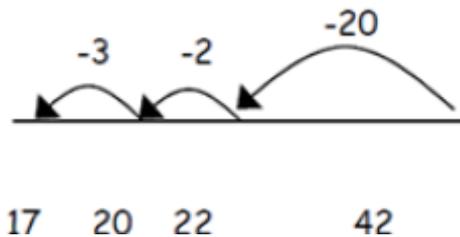
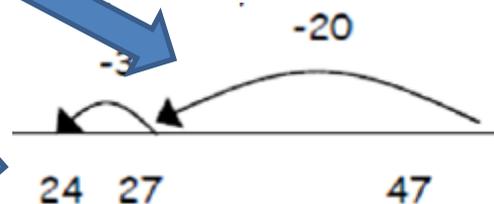
Focus: Subtracting with 2 digit numbers

In year 2 children will start to use blank numberlines to subtract by counting back which will greatly support the development of mental subtraction skills. Base 10 is also a super subtraction tool and should be used alongside blank numberline methods.



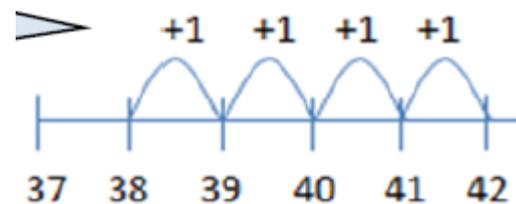
For $47 - 23 = 24$, children should start by partitioning the tens number and subtracting that first by counting back in tens. They will then subtract the units number and subtract that by counting back in 1s.

Once confident with efficient jumps, children are ready to subtract by bridging through 10, again partitioning is very important here and the children will need to be very confident with partitioning in different ways.



Once children develop their confidence of counting back they will be able to select more efficient jumps to solve a problem and will not have to partition the tens and units numbers separately.

Counting on as a mental method
Counting on is a super mental method! It is especially useful for finding the difference problems and numbers that are close together. It is important that children understand that although they are counting on, they are finding the difference which is subtraction!



Key Vocabulary

Equal to, take, take away, less, minus, subtract, leaves, distance between, difference between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is_?, *count on*, *strategy*, *partition*, *tens*, *units*

Key Skills

- ☑ Recognise the place value of each digit in a 2 digit number.
- ☑ Recall and use subtraction facts to 20 fluently, use to derive related facts to 100.
- ☑ Subtract using objects, images, 100 squares and mentally including a two digit number and ones, a two digit number and 10s and two 2 digit numbers.
- ☑ Understand and show that subtraction calculations cannot be done in any order.
- ☑ Use the inverse relationship between + and - to check calculations and solve missing number problems.
- ☑ Solve simple subtraction problems in context using written and mental methods.
- ☑ Read and write numbers to at least 100 in numerals and words.

Subtraction

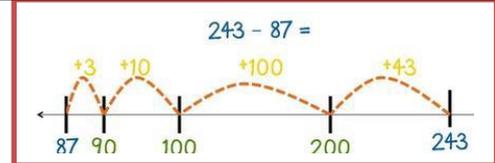
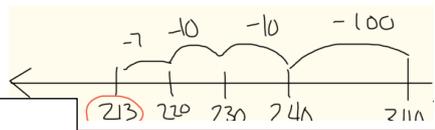
Year Three

Focus: Subtracting with 2 and 3 digit numbers

Children will consolidate their knowledge of counting back and counting on using a blank numberline to subtract. They will use these methods both written and mentally. Once children become fully confident they will be ready to move on to the partitioning column method of subtraction.

Counting on will also be used for problems greater than 100 using efficient jumps, the use of 100 square can support children's understanding of this method.

Children will continue to subtract on a numberline using efficient jumps and now apply these to 3 digit number problems. Here is an efficient example of $340 - 127 =$



Children will now have the mental skills required to approach the partitioning column method of subtraction. At first they should attempt this where no exchanging is required. Here is an example for $89 - 35 = 54$

$$\begin{array}{r} 80 + 9 \\ - 30 + 5 \\ \hline 50 + 4 \end{array}$$

Base 10 blocks representing 70 + 2. There are seven tens rods and two units cubes. To the right, the calculation is shown: $\begin{array}{r} 70 + 2 \\ - 40 + 7 \\ \hline 20 + 5 = 25 \end{array}$

Through practical subtraction children should be introduced to exchanging. Base 10 is a vital tool here as is a solid grounding with partitioning in different ways. It is important children realize that the value has not changed, we have just partitioned in a different way. As you can see here for $72 - 47$, before subtracting 7 units, a tens rod will need to be exchanged for 10 units.

Children who are secure with the concept of 'exchanging' should now be able to use the partitioning column method to subtract any 2

$$\begin{array}{r} 238 - 146 = 92 \\ \begin{array}{r} 100 \\ 200 + 30 + 8 \\ - 100 + 40 + 6 \\ \hline 0 + 90 + 2 \end{array} \end{array}$$

Key Vocabulary

Equal to, take, take away, less, minus, subtract, leaves, distance between, difference between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is_?, count on, strategy, partition, tens, units, *exchange*, *decrease*, *hundreds*, *value*, *digit*

Key Skills

- Subtract mentally: a 3 digit number and 1s, a 3 digit number and 10s and a 3 digit number and 100s.
- Estimate answers and use the inverse to check.
- Solve problems in different contexts, including missing number problems.
- Find 10 or 100 more or less than a given number.
- Recognise the place value in a 3 digit number, 100s, 10s and 1s.
- Solving finding the difference problems using counting on.
- Reading and writing numbers up to 1000 in numerals and words.
- Practise and develop mental strategies including subtracting near multiples of 10 and adjusting, counting on etc.

Subtraction

Year Four

Focus: Subtracting with numbers up to 4 digits

Children will consolidate their knowledge of the partitioning column method for subtraction with 4 digit numbers including those where exchanging is required. Once they are secure with this they will move on to the compact (traditional) method of column subtraction.

Children will consolidate their learning of the partitioning column method of subtraction and exchanging by solving calculations with more complex numbers. Place value counters will come in handy here when building children's confidence. Money can also be partitioned for subtraction e.g. £1 + 30 + 5 - £1 + 10 + 2 =

$$\begin{array}{r} 2754 - 1562 = 1192 \\ 2000 + \overset{600}{\cancel{700}} + 50 + 4 \\ - 1000 + 500 + 60 + 2 \\ \hline 1000 + 100 + 90 + 2 \end{array}$$

Once confident children are ready to move on to the compact method of subtraction. Encourage children to complete a calculation in the partitioning column methods and then model compact method. See if children can see how they are linked and discuss which is simpler. Although this is seen as the 'easiest' method it does not mean that it is necessarily the best method and they need to carefully select the best method for the problem they are solving.

$$\begin{array}{r} \overset{6}{2} \overset{1}{\cancel{7}} 5 4 \\ - 1 5 6 2 \\ \hline 1 1 9 2 \end{array}$$

Key Vocabulary

Equal to, take, take away, less, minus, subtract, leaves, distance between, difference between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is_?, count on, strategy, partition, tens, units, exchange, decrease, hundreds, value, digit, *inverse*.

Key Skills

- Subtract by counting on where numbers are close together or they are near to multiples of 10, 100 etc.
- Children select a mental, written or jotting method depending on what the problem requires.
- Children estimate and use the inverse operation to check a problem.
- Children solve 2 step problems involving + and -, picking the correct operation and method.
- Children solve simple money and measure problems with fractions and decimals.
- Find 1000 more or 1000 less than a given number.
- Count backwards through zero including negative numbers.
- Recognise the place value of each digit in a 4 digit number.
- Round any number to the nearest 10, 100 or 1000.
- Solve number and practical problems that involve increasingly large positive integers.

Subtraction

Year Five

Focus: Subtracting with numbers beyond 4 digits including decimals

Children in year 5 will continue to use the compact column method of subtraction to solve problems including those where exchanging is required. They will subtract larger integers and begin to subtract decimal amounts.

Children will come across problems where exchanging will need to take place several times to complete the problem.

	2 ²	1 ¹⁰	0 ¹⁰	5 ⁶	6 ⁶
-		2	1	2	8
<hr/>					
	2	8	9	2	8

Where there is a space in a column it is important that children add a zero so they understand the value and know what to subtract in that column.

Once confident with large integers, children will now be ready to move onto decimal numbers including lots in the context of measures and money. Just like addition, it is important that the children line up the decimal point and understand why they are doing this.

	6 ⁶	7 ¹⁰	6 ⁶	9 ⁸	.	0 ⁰
-		3	7	2	.	5
<hr/>						
	6	7	9	6	.	5

Key Vocabulary

Equal to, take, take away, less, minus, subtract, leaves, distance between, difference between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is_?, count on, strategy, partition, tens, units, exchange, decrease, hundreds, value, digit, inverse, *tenths*, *hundredths*, *decimal place*, *decimal*

Key Skills

- Subtract mentally with increasingly large numbers.
- Use rounding and estimation to check answers to calculations.
- Solve addition and subtraction multi step problems, deciding which operations to use and why.
- Read, write, order and compare numbers to at least 1 million and understand the value of each digit.
- Count forwards or backwards in steps of powers of 10 up to 1 million.
- Understand negative numbers in context and count forwards and backwards through 0.
- Round any number up to 1 million to the nearest 10, 100, 1000, 10 000 and 100 000.

Subtraction

Year Six

Focus: Subtracting with increasingly complex numbers including decimals

In year 6, children need to use mental methods and the compact column method of subtraction to solve an increasingly complex range of calculation including those with integers, those with decimals and those with mixed numbers.

$$\begin{array}{r} \overset{\circ}{9} \overset{\circ}{6} 99 \\ - 89949 \\ \hline 60750 \end{array}$$

Children will use the compact method to solve problems involving integers up to 6 digits and beyond and solve problems where they will need to use 'exchanging' several times.

They will also solve problems in context involving increasingly large decimals. They will need to continue using their knowledge of decimal points to line up their numbers and place zeroes in any empty places so they fully understand the value of that column.

$$\begin{array}{r} \overset{\circ}{9} \overset{\circ}{1} 5.3419 \\ - 36.080 \\ \hline 69.339 \end{array}$$

Key Vocabulary

Equal to, take, take away, less, minus, subtract, leaves, distance between, difference between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is _?, count on, strategy, partition, tens, units, exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal place

Key Skills

- Solve addition and subtraction multi step problems in context, deciding which operations to use and why.
- Read, write, order and compare numbers to at least 10 million and understand the value of each digit.
- Round any whole number up to 10 million to the nearest 10, 100, 1000, 10 000, 100 000, or 1 million.
- Use negative numbers in context and calculate intervals across zero.
- Look at a calculation and decide whether you need to use a mental method, a jotting, a written method or a calculator to solve.

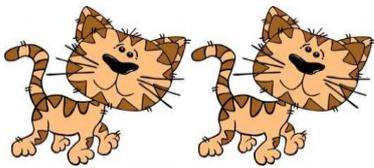
Multiplication

Year One

Focus: Repeated addition with objects, arrays and pictorial representations.

In year one children will be exposed to many different multiplication based activities in a variety of contexts. Much of this will be repeated addition activities or be linked to counting in 2s, 5s or 10s.

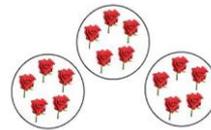
How many legs will 2 cats have?



$$4 + 4 = 8$$

Children use images and pictorial representations to solve simple problems that involve repeated addition. They may wish to use the picture to support or use other equipment. Adult support at this stage is to be expected. Some children may start to see the link between the problem below and counting in 5s and be able to use mental skills to solve the problem.

There are 5 roses in each garden. How many roses in 3 gardens?



$$5 + 5 + 5 = 15$$

Key Vocabulary

Groups of, lots of, times, array, altogether, multiply, count

Key Skills

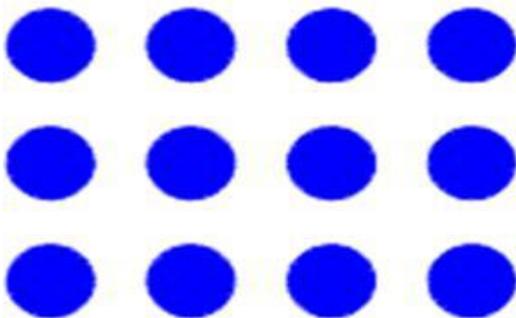
- Count in multiples of 2, 5 and 10.
- Solve 1 step problems involving multiplication using objects, arrays or pictures with support.
- Make connections between arrays and counting in 2s, 5s and 10s.
- Begin to understand doubling using objects and pictorial representations.
- Solve practical problem solving activities counting equal sets or groups.
- Have lots of practice counting and bundling groups of objects into 2s, 5s and 10s.

Multiplication

Year Two

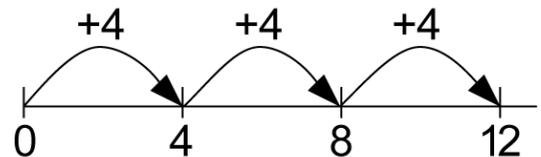
Focus: Multiplying using arrays and repeated addition- 2,3,4,5,10x table facts

In year 2 children will be aware of simple arrays and pictorial representations and understand what they mean. In year 2 children will develop the knowledge of how to make their own arrays to solve a problem and also how repeated addition on a numberline can get them to a solution.

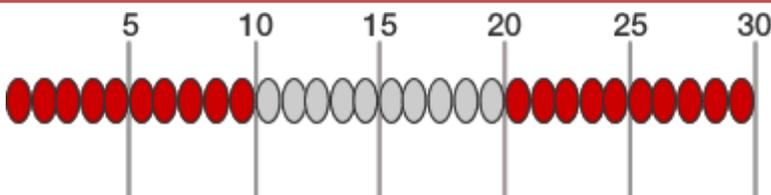


$$3 \times 4 = 12$$
$$3 \times 4 = 4 + 4 + 4 = 12$$
$$4 \times 3 = 3 + 3 + 3 + 3 = 12$$

Arrays are super for children to solve the answer to simple problems. They are also great for showing children the commutative law, for example, if you turned this array for $3 \times 4 = 12$ sideways you would see that 4×3 also equals 12.



Repeated addition is a good progression from arrays. It encourages the children to use addition facts on a blank numberline and count up to their answer as shown on the example above which models that $3 \times 4 = 12$.



Mental methods and practical apparatus are still very important at this stage. Visual images such as the bead string to the left that demonstrates $6 \times 5 = 30$ will support children's visualization of multiplication and allow them to develop stronger mental skills.

Key Vocabulary

Groups of, lots of, times, array, altogether, multiply, count, *multiplied by*, *repeated addition*, *column*, *row*, *commutative*, *sets of*, *equal groups*, *times as big as*, *once*, *twice*, *three times*

Key Skills

- Count in steps of 2,3 and 5 from zero and in 10s from any number.
- Recall and use multiplication facts for the 2,5 AND 10 times tables.
- Recognise odd and even numbers.
- Write and calculate number statements using the x and = signs.
- Show that multiplication can be done in any order (the commutative law).
- Solve a range of multiplication problems using objects, arrays, repeated addition, mental methods and multiplication facts.
- Use and become familiar with all of the above multiplication language.

Multiplication

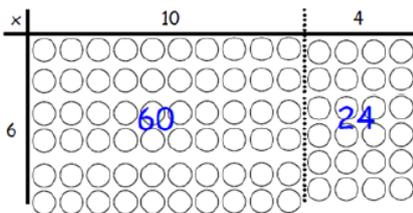
Year Three

Focus: Multiplying 2 digit numbers by 1 digit numbers

In year 3 children will move on from arrays and start using the grid method of multiplication. It is essential that before children move onto the grid method they are completely confident with all previous methods and have a solid grounding with mental methods and partitioning.

Before moving on to grid multiplication, children need to be able to ...

- Partition numbers into tens and units
- Multiply single digits by multiples of 10 ($3 \times 30 = 90$).
- Quickly recall multiplication facts for the 2,3,4,5,8 and 10 x tables.
- Use any previous method to work out unknown multiplication facts, quickly and accurately.



The grid method should be introduced using an arrays model such as the one to the left for 14×6 . Children need to use their partitioning skills to partition the two digit number and then use their existing knowledge of arrays to come to an answer with minimal support.

Multiplication grid method requires good organization but also a solid understanding of partitioning and multiplication facts, as you can see in the example to the right for 35×7 . The children need to remember that once they have multiplied the partitioned parts of the number, they then need to add the two.

x	30	5
7	210	35

$$210 + 35 = 245$$

Key Vocabulary

Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times, *partition*, *grid method*, *multiple*, *product*, *tens*, *units*, *value* .

Key Skills

- Recall and use multiplication facts for the 2,3,4,5,8 and 10 multiplication tables and multiply multiples of 10.
- Write and calculate number sentences using known x tables.
- Answer 2 digit x 1 digit problems using mental and written methods.
- Solve multiplication problems in context including missing number problems.
- Develop mental strategies using commutativity (e.g. $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$) and for missing number problems.

Multiplication

Year Four

Focus: Multiplying 2 and 3 digit numbers by 1 digit numbers

In year 4 children need to use the grid method confidently to solve problems where a 2 or 3 digit number is multiplied by a one digit number. They need to move on to the use of short multiplication to solve 3 digit number multiplied by 1 digit problems.

x	600	10	3
5	3000	50	15

Add up 3000, 50 and 15 to make 3065.

$$613 \times 5 = 3065$$

The grid method is extended in year 4 so children will now multiply 3 digit numbers by 1 digit numbers. When adding the 3 answers up to create a total, column addition could be used to ensure accuracy, especially where bridging will be needed.

The compact 'short multiplication' method is tricky and needs to be approached carefully. At first children should solve a problem using grid method and then observe the teacher solve a problem using short multiplication and make comparisons. How are they similar? Children need to go through it very slowly and carefully, unpicking each step until they are fully confident.

$$\begin{array}{r} 463 \\ \times \quad 8 \\ \hline 3704 \\ \hline 52 \end{array}$$

It is at this stage that approximation and estimation should become a regular part of classroom practice. Children should approximate an answer before using a method so they know if their answer is accurate or not.

$$253 \times 9 \text{ is approximately } 250 \times 10 = 2500$$

Key Vocabulary

Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times, partition, grid method, multiple, product, tens, units, value, *inverse*

Key Skills

- Count in multiples of 6,7,8,9,25 and 1000.
- Recall multiplication facts for all multiplication tables up to 12 x 12.
- Recognise place value of digits in up to 4 digit numbers.
- Multiply large numbers and multiple values mentally using place value, known facts and derived facts.
- Use commutativity mentally to solve problems.
- Solve problems in a range of contexts that are increasingly complex.

Multiplication

Year Five

Focus: Multiplying up to 4 digits by 1 or 2 digits

In year 5 children will continue to use short multiplication to solve increasingly richer problems that involve multiplying by 1 digit. They will then move on to long multiplication for problems that involve multiplying by 2 digits. Approximation will play an important part- with children making approximations before using long multiplication to help check their answer is correct.

	3	6	5	2	
x				8	
<hr/>					
2	9	2	1	6	
	5	4			

Children will use short multiplication in a range of increasingly challenging problems. Solving using the grid method and then comparing to the short multiplication method will help cement the children's understanding.

	10	8	
10	100	80	
3	30	24	

When multiplying by more than 1 digit, children need to use long multiplication. Like with short multiplication, they will solve the problem using the grid method first and then make comparisons until their understanding is secure. In the example below the top row shows 18×3 and the bottom shows 18×10 . The final row shows the total of both calculations.

	1	2	3	4	
x			1	6	
<hr/>					
	7	4	0	4	
1	2	3	4	0	
<hr/>					
1	9	7	4	4	

Once long multiplication methods are secure, children are ready to move on to more challenging problems which require greater levels of mental calculation. The problem to the left shows 1234×6 on the top line, 1234×10 on the bottom line and the total of both calculations on the final row.

		1	8	
x		1	3	
<hr/>				
		5	4	
			2	
	1	8	0	
<hr/>				
	2	3	4	

Key Vocabulary

Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times, partition, grid method, multiple, product, tens, units, value, inverse, *square*, *factor*, *integer*, *decimal*, *short/long multiplication*, *carry*.

Key Skills

- Identify multiples and factors, using secure x table facts to 12×12 .
- Solve problems where larger numbers are decomposed into their factors.
- Multiply and divide integers and decimals by 10, 100 and 1000.
- Recognise and use square and cube numbers and their notation.
- Solve problems that have different combinations of operations, picking the most useful methods.

Multiplication

Year Six

Focus: Consolidating short and long multiplication, multiplying decimals by 1 digit

In year 6 children will consolidate all they know about short and long multiplication before they go to Secondary school. They will also learn the new skill of using short multiplication to multiply decimal numbers to 2 decimal places.

A handwritten multiplication problem on a blue grid. The first number is 3.19 and the second number is 8. A horizontal line is drawn under the 8. Below the line, the product 25.52 is written. The decimal point in the product is aligned with the decimal point in the first number. The grid lines help align the digits correctly.

When multiplying decimals it is important to remember that the digit you are multiplying by needs to be lined up with the ones digits. As with all decimal work, the decimal points must be lined up and the children need to have a clear understanding why that is.

Key Vocabulary

Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times, partition, grid method, multiple, product, tens, units, value, inverse, square, factor, integer, decimal, short/long multiplication, carry, *tenths*, *hundredths*, *decimals*.

Key Skills

- Multiply up to 4 digits by 2 digits using long multiplication.
- Solve mixed operation and large number problems using mental methods.
- Solve multi step problems involving a range of operations.
- Estimate and approximate answers of problems to improve accuracy.
- Round any integer to the determined level of accuracy.

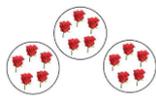
Division

Year One

Focus: Grouping and sharing small quantities without remainders

As an introduction to division, children in year 1 will solve problems in familiar and relevant contexts where they have to group and share. They will use objects and pictorial representations to solve problems and they will begin to use counting in 2s, 5s and 10s to support their problems solving.

A farmer has 15 roses and shares them between 3 friends. How many roses do they each get?



15 roses shared between 3 = 5 roses each

Children need to learn grouping and sharing alongside each other so they understand how they are linked. Grouping will also help children understand how multiplication can be used to solve division problems. Contextual problems will strengthen children's understanding of division.

Bats fly in groups of 2. How many groups of 2 will there be if there are 8 bats?



8 bats shared into groups of 2 = 2 bats in each group.

Key Vocabulary

Share, share equally, one each, two each..., group, groups of, lots of, array.

Key Skills

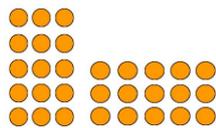
- Solve one step problems involving multiplication and division using concrete objects with support from adults.
- Children use grouping and sharing to understand division and to begin to understand finding simple fractions.
- Children make connections between arrays and counting in 2s, 5s and 10s.
- Children use halving and understand that this is the same as sharing into 2 equal groups.

Division

Year Two

Focus: Grouping and sharing larger quantities using written methods and symbols
Children will continue to use the methods of sharing and grouping in division with objects to support their understanding of arrays for sharing and grouping and the division numberline for grouping.

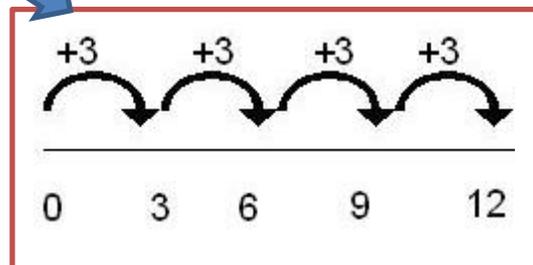
To solve problems such as $15 \div 3 =$, children will share 15 objects into 3 groups like in the first array or make groups of 3 until they get to 15, like in the second image.



Completing both of these processes will help children see the link between sharing and grouping but also the link between $15 \div 3 = 5$ and $15 \div 5 = 3$.

The difference between grouping and sharing should be discussed regularly and visual models and diagrams are very important. Children should solve a variety of contextual problems that will require them to group or share.

Children will start to group on a numberline- which will help cement their understanding of division as grouping. When grouping on a numberline, children will start with a zero at the beginning and will write the dividend at the end of the line, they will then jump in steps of the divisor. The example to the right shows a numberline for the calculation $12 \div 3 = 4$ as there were 4 jumps of 3 to get to 12.



Key Vocabulary

Share, share equally, one each, two each..., group, groups of, lots of, array, divide, *divided by*, *divided into*, *division*, *grouping*, *number line*, *left*, *left over*.

Key Skills

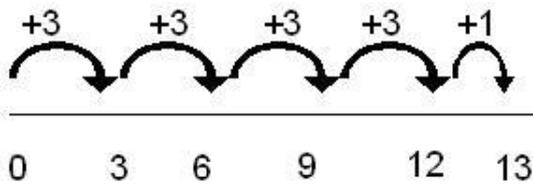
- Count in steps of 2,3 and 5 from 0.
- Recall and use \times and \div facts for the 2,5 and 10 times tables.
- Solve division problems and write division number sentences for problems.
- Understand that division is not commutative unlike multiplication.
- Solve increasingly challenging division problems using concrete objects, arrays, and simple written methods such as grouping on a numberline.

Division

Year Three

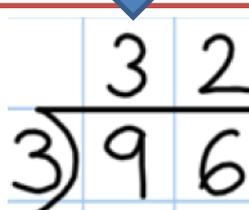
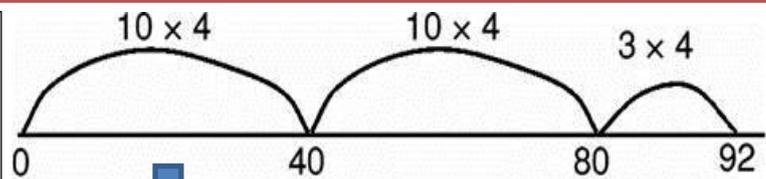
Focus: Dividing 2 digit numbers by 1 digit numbers moving from numberline methods to short division

Children in year 3 will continue to use a numberline to solve division problems and will begin to jump more than one step at a time in the style of 'chunking'. Once confident they will move on to short division without any remainders.



Children will begin to use the grouping numberline method to solve problems with remainders. They will start on zero and write the dividend at the end of their numberline. They will jump in steps of the divisor until they get as close to the end as possible. Whatever is left over is the remainder. Using cubes or arrays alongside the numberline will consolidate understanding.

Once confident children will begin to solve problems on a grouping numberline involving bigger numbers. To solve this effectively they will need to subtract chunks of the divisor. As you can see in the image for $92 \div 4$, a step of 10 groups of 4 has been jumped, followed by another step of 10 jumps, finally followed by a step of 3 jumps of 4. This means that in total 4 was jumped 23 times making 23 the answer.



Once children are confident with numberline methods then they should start work on short division. First of all arrays should be used to show a division calculation, the same calculation should then be shown in the short multiplication method. Place value should be regularly discussed so children realize that they are partitioning the dividend and dividing the units then the tens by the divisor.

Initially children will start with simple problems where each digit is a multiple of the divisor.

Key Vocabulary

Share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, *inverse*, *short division*, *carry*, *remainder*, *multiple*.

Key Skills

- Recall and use \times and \div facts for the 2,3,4,5,6,8 and 10 x tables (using doubling to connect the 2,4 and 8 x tables)
- Solving division problems where a 2 digit number is divided by a 1 digit number using mental and written.
- Solve problems in a variety of contexts including missing number problems.
- Pupils begin to derive related facts e.g. $9 \div 3 = 3$ means $90 \div 3 = 30$ or $90 \div 30 = 3$.
- Pupils develop confidence in written methods, moving from numberlines to short division.

Division

Year Four

Focus: Consolidating and extending use of short division

Children in year 4 will continue to use short division to solve division problems. They will begin to work on remainders, including problems where there are remainders in the first numbers but not in the final answer.

$$\begin{array}{r} 12 \\ 8 \overline{) 96} \end{array}$$

Once confident with the method of short division, they will move on to problems where the first digit of the dividend is not a multiple of the divisor and therefore a remainder will need to be carried. Children may need to use other equipment to calculate the division and multiplication facts required.

$$\begin{array}{r} 218 \\ 4 \overline{) 872} \end{array}$$

$$\begin{array}{r} 035 \\ 5 \overline{) 175} \end{array}$$

Children who can use short multiplication problems with remainders (but not those in the final answer) are now ready to work on 3 digit problems. Again, there should be remainders in the calculation but never in the final answer.

Once children are confident at dividing with 3 digits, they need to attempt problems where the answer in the first column (hundreds column) is a zero. They may wish to record the hundred initially as this will help them remember its place and the numbers value.

Key Vocabulary

Share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple, *divisible by*, *factor*.

Key Skills

- Recall multiplication and division facts for all numbers to 12 x 12.
- Use place value and known facts to derive facts mentally- including multiplying and dividing by 100, 10 and 1.
- Practise mental methods and extend this to three digit numbers using derived facts- e.g. $100 \div 5 = 20$ so $20 \times 5 = 100$.
- Solve two step problems with increasingly harder numbers in a range of contexts, using language to identify the correct operation.
- Correspondence problems should be introduced such as 3 cakes are shared equally between 10 children, 1 man has 6 cats so how many cats do 3 men have etc.

Division

Year Five

Focus: Extending use of short multiplication to 4 digits and remainders

Children in year 5 will use short division to solve problems up to 4 digits long. For the first time they will use short division to solve problems that have a remainder in the final answer.

A handwritten short division problem on a grid background. The divisor is 8, and the dividend is 53029. The quotient is 0663 with a remainder of 5. The numbers are written in a cursive style.

In year 5 children will begin to solve division problems where a number up to 4 digits is divided by a single digit number including answers with remainders. These division problems need to be contextual so the children learn how to express the remainder- as a number, a fraction, a decimals, rounded up or rounded down.

Key Vocabulary

Share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple, divisible by, factor, *quotient, prime number, prime factors, composite number (non-prime)*.

Key Skills

- Multiply and divide numbers mentally, using known facts.
- Identify multiples and factors, including all factor pairs of a number and common factors between 2 numbers.
- Solve \times and \div problems where larger numbers are decomposed into their factors.
- Multiply and divide whole numbers and decimals by 10, 100 and 1000.
- Use vocabulary of prime numbers, prime factors and composite numbers.
- Work out whether a number up to 100 is prime and know all prime numbers to 30.
- Use and understand multiplication and division as inverses.
- Present division with remainders answers differently, showing the remainder as a fraction, decimal or whole number by rounding.
- Solve problems with a combination of all four operations including fraction scaling problems and problems involving simple rates.

Division

Year Six

Focus: Using short division to divide 4 digit numbers and express remainders as decimals and long division for dividing 2 digit numbers

In year 6, children will use short division to divide decimal numbers by single digit numbers. The final step of division will be long division which will be used to divide numbers by 2 digits.

$$\begin{array}{r} 0812.125 \\ 8 \overline{)6497.000} \end{array}$$

The focus in year 6 is not so much the method of short division but how the remainders are expressed- children need to express remainders as decimals and fractions- depending on the context of the question.

The remainder in this answer would have been 1 but it has been expressed as a decimal. To do this, children need to insert a decimal point next to the units and carry the remainder over the decimal point. Zeroes are inserted to the right of the decimal point to show that there was no value.

A great way of remembering the steps of long division is **Does McDonalds Serve Burgers?**



To divide by 2 digit numbers, the children will use the method of long division. The example to the right clearly shows the method in the 'Burger' steps, where as the example to the left shows what a completed method would look like. Any remainders would need to be expressed in a way that matched the context of the problem.

$$\begin{array}{r} 291 \\ 45 \overline{)13095} \\ \underline{90} \\ 409 \\ \underline{405} \\ 45 \\ \underline{45} \\ 0 \end{array}$$

Divide:

$$\begin{array}{r} 2 \\ 3 \overline{)75} \\ \underline{6} \\ 15 \end{array}$$

3 goes into 7 2 times... with some extra

Multiply:

$$\begin{array}{r} 2 \\ 3 \overline{)75} \\ \underline{6} \\ 15 \end{array}$$

$2 \times 3 = 6$

Subtract:

$$\begin{array}{r} 2 \\ 3 \overline{)75} \\ \underline{6} \\ 15 \end{array}$$

Bring Down:

$$\begin{array}{r} 2 \\ 3 \overline{)75} \\ \underline{6} \\ 15 \end{array}$$

Repeat:

$$\begin{array}{r} 25 \\ 3 \overline{)75} \\ \underline{6} \\ 15 \\ \underline{15} \\ 0 \end{array}$$

$15 \div 3 = 5$
 $5 \times 3 = 15$

Key Vocabulary

Share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple, divisible by, factor, quotient, prime number, prime factors, composite number (non-prime), *common factor*

Key Skills

- ☑ Use multiplication and division facts up to 12×12 to solve more complex problems.
- ☑ Decide when to use short or long division and interpret remainders in a way that is appropriate to the problem.
- ☑ Perform mental calculations for problems involving large numbers and mixed calculations.
- ☑ Identify common factors, common multiples and prime numbers.
- ☑ Use estimation to check answers to calculations and determine accuracy.
- ☑ Use written methods of division to solve decimal problems up to 2 decimal places.
- ☑ Solve problems which require rounding to 10, 100, 1000 and beyond.

